1. If $x=\sqrt[4]{x^3+6x^2}$, then the sum of all possible solutions for x is:

A. -2
B. 0
C. 1
D. 3
E. 5

Take the given expression to the 4th power: $x^4=x^3+6x^2$;

Re-arrange and factor out x^2: $x^2(x^2-x-6)=0$;

Factorize: $x^2(x-3)(x+2)=0$;

So, the roots are $x=0$, $x=3$ and $x=-2$. But x cannot be negative as it equals to the even (4th) root of some expression ($\sqrt{expression}\geq 0$), thus only two solution are valid $x=0$ and $x=3$.

The sum of all possible solutions for x is 0+3=3.

Answer: D.

2. The equation x^2 + ax - b = 0 has equal roots, and one of the roots of the equation x^2 + ax + 15 = 0 is 3. What is the value of b?

A. -64
B. -16
C. -15
D. -1/16
E. -1/64

Since one of the roots of the equation $x^2+ax+15=0$ is 3, then substituting we'll get: $3^3+3a+15=0$. Solving for a gives $a=-8$.

Substitute $a=-8$ in the first equation: $x^2-8x-b=0$.

Now, we know that it has equal roots thus its discriminant must equal to zero: $d=(-8)^2+4b=0$. Solving for b gives $b=-16$.

Answer: B.

3. If a and b are positive numbers, such that a^2 + b^2 = m and a^2 - b^2 = n, then ab in terms of m and n equals to:

A. $\frac{\sqrt{m-n}}{2}$
B. $\frac{\sqrt{mn}}{2}$
C. $\frac{\sqrt{m^2-n^2}}{2}$
D. $\frac{\sqrt{n^2-m^2}}{2}$
E. $\frac{\sqrt{m^2+n^2}}{2}$

Sum the two equations: $2a^2=m+n$;
Subtract the two equations: $2b^2=m-n$;

Multiply: $4a^2b^2=m^2-n^2$;

Solve for ab: $ab=\frac{\sqrt{m^2-n^2}}{2}$

Answer: C.

4. What is the maximum value of -3x^2 + 12x -2y^2 - 12y - 39 ?

A. -39
B. -9
C. 0
D. 9
E. 39

$$-3x^2+12x-2y^2-12y-39=-3x^2+12x-12-2y^2-12y-18-9=-3(x^2-4x+4)-2(y^2+6y+9)-$$

$$=-3(x-2)^2-2(y+3)^2-9.$$

So, we need to maximize the value of $-3(x-2)^2-2(y+3)^2-9$.

Since, the maximum value of $-3(x-2)^2$ and $-2(y+3)^2$ is zero, then the maximum value of the whole expression is $0+0-9=-9$.

Answer: B.

5. If x^2 + 2x -15 = -m, where x is an integer from -10 and 10, inclusive, what is the probability that m is greater than zero?

A. 2/7
B. 1/3
C. 7/20
D. 2/5
E. 3/7

Re-arrange the given equation: $-x^2-2x+15=m$.

Given that x is an integer from -10 and 10, inclusive (21 values) we need to find the probability that $-x^2-2x+15$ is greater than zero, so the probability that $-x^2-2x+15>0$.

Factorize: $(x+5)(3-x)>0$. This equation holds true for $-5<x<3$.

Since x is an integer then it can take the following 7 values: -4, -3, -2, -1, 0, 1, and 2.

So, the probability is 7/21=1/3.

Answer: B.

6. If mn does not equal to zero, and m^2n^2 + mn = 12, then m could be:

I. -4/n
II. 2/n
III. 3/n

A. I only
B. II only
C. III only
D. I and II only
E. I and III only

Re-arrange: $(mn)^2+mn-12=0$.

Factorize for mn: $(mn+4)(mn-3)=0$. Thus $mn=-4$ or $mn=3$.

So, we have that $m=-\frac{4}{n}$ or $m=\frac{3}{n}$.

Answer: E.

7. If x^4 = 29x^2 - 100, then which of the following is NOT a product of three possible values of x?

I. -50
II. 25
III. 50

A. I only
B. II only
C. III only
D. I and II only
E. I and III only

Re-arrange and factor for x^2: $(x^2-25)(x^2-4)=0$.

So, we have that $x=5$, $x=-5$, $x=2$, or $x=-2$.

$-50=5*(-5)*2$;
$50=5*(-5)*(-2)$.

Only 25 is NOT a product of *three* possible values of x

Answer: B.

8. If m is a negative integer and m^3 + 380 = 381m , then what is the value of m?

A. -21
B. -20
C. -19
D. -1
E. None of the above

Given $m^3+380=380m+m$.

Re-arrange: $m^3-m=380m-380$.

$m(m+1)(m-1)=380(m-1)$. Since m is a negative integer, then $m-1\neq0$ and we can safely reduce by $m-1$ to get $m(m+1)=380$.

So, we have that 380 is the product of two consecutive *negative* integers: $380=-20*(-19)$, hence $m=-20$.

Answer: B.

9. If $x=(\sqrt{5}-\sqrt{7})^2$, then the best approximation of x is:

A. 0
B. 1
C. 2
D. 3
E. 4

$x=(\sqrt{5}-\sqrt{7})^2=5-2\sqrt{35}+7=12-2\sqrt{35}$.

Since $\sqrt{35}\approx6$, then $12-2\sqrt{35}\approx12-2*6=0$.

Answer: A.

10. If f(x) = 2x - 1 and g(x) = x^2, then what is the product of all values of n for which f(n^2)=g(n+12) ?

A. -145
B. -24
C. 24
D. 145
E. None of the above

$f(x)=2x-1$, hence $f(n^2)=2n^2-1$.
$g(x)=x^2$, hence $g(n+12)=(n+12)^2=n^2+24n+144$.

Since given that $f(n^2)=g(n+12)$, then $2n^2-1=n^2+24n+144$. Re-arranging gives $n^2-24n-145=0$.

Next, **Viete's theorem states that for the roots** x_1 **and** x_2 **of a quadratic equation** $ax^2+bx+c=0$:

$x_1+x_2=\frac{-b}{a}$ **AND** $x_1*x_2=\frac{c}{a}$.

Thus according to the above $n_1*n_2=-145$.

Answer: A.